Use an appropriate linear approximation to estimate $\csc 0.5$.

SCORE: _____ / 5 PTS

$f(x) = \csc x$ NEAR $x = \frac{\pi}{6}$ $\quad f'(x) = \underbrace{-\csc x \cot x}_{1}$

$$f(x) \approx f\left(\tfrac{\pi}{6}\right) + f'\left(\tfrac{\pi}{6}\right)\left(x - \tfrac{\pi}{6}\right)$$

$$= \csc\tfrac{\pi}{6} - \left(\csc\tfrac{\pi}{6}\cot\tfrac{\pi}{6}\right)\left(x - \tfrac{\pi}{6}\right)$$

$$= 2 - 2\sqrt{3}\left(x - \tfrac{\pi}{6}\right)$$

$$\csc 0.5 \approx \underbrace{2}_{1} \underbrace{- 2\sqrt{3}}_{1\frac{1}{2}} \underbrace{\left(\tfrac{1}{2} - \tfrac{\pi}{6}\right)}_{1\frac{1}{2}} = 2 - \sqrt{3} + \frac{\pi\sqrt{3}}{3}$$

A rock sits on a straight shoreline at the point closest to a lighthouse. **SCORE: _____ / 10 PTS**

The lighthouse is located on a small island 2 km from the shoreline, and revolves at a constant rate, casting a spot of light onto the shoreline.

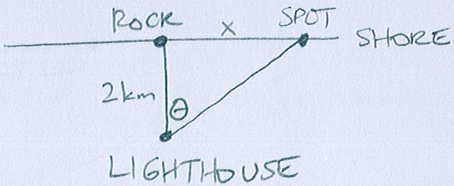
At the moment when the spot of light is 3 km from the rock, the spot is moving along the shoreline at 4 km per second.

How quickly is the lighthouse revolving ?

<u>You must state/show clearly what each variable you use represents.</u>

<u>You must show the units during the intermediate steps of your work, and you must state the units for the final answer.</u>

1

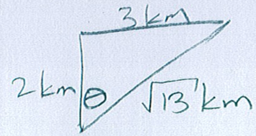


$$\frac{dx}{dt} = 4 \text{ km/s}$$

WANT $\left.\frac{d\theta}{dt}\right|_{x = 3\text{ km}}$

2 $\boxed{\tan\theta = \frac{x}{2\text{ km}}}$

WHEN $x = 3$ km



$$2\tan\theta \text{ km} = x$$

2 $\boxed{2\sec^2\theta \frac{d\theta}{dt} \text{ km} = \frac{dx}{dt}}$

$$\sec^2\theta = \left(\frac{\sqrt{13}}{2}\right)^2 = \frac{13}{4}$$

1

2 $\boxed{\cancel{2}\left(\frac{13}{4}\right)\frac{d\theta}{dt}\,\cancel{\text{km}} = \overset{2}{\cancel{4}}\,\cancel{\text{km}}/s}$

$$\frac{d\theta}{dt} = \frac{8}{13 s} \text{ OR } \frac{8}{13} \text{ RADIANS PER SECOND}$$

1 1 ONLY IF UNITS SHOWN IN EQUATION

THE LIGHTHOUSE IS REVOLVING AT $\boxed{\frac{8}{13}}$ $\boxed{\text{RADIANS PER SECOND}}$

Prove the derivative of $\operatorname{sech} x$ using the known derivative of $\cosh x$, along with the quotient rule. SCORE: _____ / 4 PTS

<u>Show all work. You must NOT use the chain rule.</u>

$$\frac{d}{dx}\operatorname{sech} x = \frac{d}{dx}\frac{1}{\cosh x}$$

$$= \frac{0 \cdot \cosh x - 1 \cdot \sinh x}{\cosh^2 x} \quad 2 \quad \tfrac{1}{2}$$

$$= \boxed{\frac{-\sinh x}{\cosh^2 x}} = \boxed{-\operatorname{sech} x \tanh x} \quad 1 \quad \tfrac{1}{2}$$

MUST HAVE NEGATIVE IN FRONT

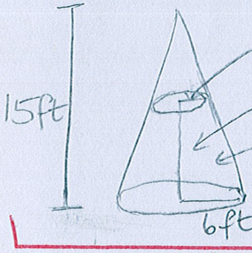
The base of a 15 foot tall conical tank has a radius of 6 feet. **SCORE: _____ / 11 PTS**

Water is entering the tank at 20π cubic feet per minute.

How quickly is the radius of the surface of the water changing when the water is 10 feet high in the tank ?

You must state/show clearly what each variable you use represents.

You must show the units during the intermediate steps of your work, and you must state the units for the final answer.

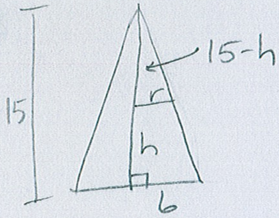


r = RADIUS OF WATER SURFACE

h = HEIGHT OF WATER

V = VOLUME OF WATER

1



$$\frac{dV}{dt} = 20\pi \text{ ft}^3/\text{min}$$

WANT $\left.\frac{dr}{dt}\right|_{h=10\text{ ft}}$

$$\frac{15\text{FT}-h}{r} = \frac{15\text{FT}}{6\text{FT}}$$

$$15\text{FT}-h = \frac{5}{2}r$$

WHEN $h = 10\text{FT}$

$$5\text{FT} = \frac{5}{2}r$$

$$2\text{FT} = r$$

$$V = \frac{1}{3}\pi(6\text{FT})^2(15\text{FT}) - \frac{1}{3}\pi r^2(15\text{FT}-h)$$

$$V = 180\pi \text{ ft}^3 - \frac{1}{3}\pi r^2\left(\frac{5}{2}r\right)$$

$$V = 180\pi \text{ ft}^3 - \frac{5}{6}\pi r^3 \quad 4$$

$$\frac{dV}{dt} = -\frac{5}{2}\pi r^2 \frac{dr}{dt} \quad 2$$

$$\cancel{20}^{\,\cancel{4}\,2}\cancel{\pi}\,\frac{\text{ft}^{\cancel{3}}}{\text{min}} = -\frac{5}{\cancel{2}}\cancel{\pi}(2\text{ ft})^2\frac{dr}{dt}$$

$$-2\,\frac{\text{ft}}{\text{min}} = \frac{dr}{dt} \quad 1\tfrac{1}{2}$$

THE RADIUS OF THE WATER'S SURFACE IS SHRINKING BY 2 FEET PER MINUTE

$1\tfrac{1}{2}$ 1 ONLY IF UNITS SHOWN IN EQUATION